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*Exclusive Nightclubs and Lonely Hearts  
Columns: Non-monotone Participation in  
Optional Intermediation*

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THE UNIVERSITY *of* EDINBURGH

# Exclusive Nightclubs and Lonely Hearts

## Columns: Non-monotone Participation in Optional Intermediation

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### Abstract

In many decentralised markets, the traders who benefit most from an exchange do not employ intermediaries even though they could easily afford them. At the same time, employing intermediaries is not worthwhile for traders who benefit little from trade. Together, these decisions amount to non-monotone participation choices in intermediation: only traders of middle “type” employ intermediaries, while the rest, the high and the low types, prefer to search for a trading partner directly. We provide a theoretical foundation for this, hitherto unexplained, phenomenon. We build a dynamic matching model, where a trader’s equilibrium bargaining share is a convex increasing function of her type. We also show that this is indeed a necessary condition for the existence of non-monotone equilibria.

**JEL Numbers:** D40, C78

**Keywords:** Two-sided markets, intermediation, dynamic matching.

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# 1 Introduction

In many decentralised markets for heterogeneous goods or services, exchange is aided by intermediaries. Even if trade is possible and legal without them, intermediaries exist because they increase the realised gains from trade by lowering the traders' search costs and by reducing the extent of mismatch. Nevertheless, not all traders use intermediaries. Traders who stand little to gain from exchange may simply find intermediation too expensive, but one often sees the traders on the two sides of the market who have the highest surplus between them decide to trade in the direct search market – risking severe mismatch and/or incurring high search expenditure – despite the availability of “cheap” intermediation. For example, as the title hints, the most desirable singles neither advertise in “lonely hearts” newspaper columns nor join on-line dating services; also, in many instances, the best jobs are not filled through agencies, the best shops are not in shopping centres, the most exclusive holidays are not on offer in travel agencies and the best properties do not come on the open market.<sup>1</sup> As further examples, some banks do not allow mortgage brokers to offer their products, and some insurers advertise the fact that they are not available on web comparison sites. In all of these markets, it is also the case that the “worst” traders do not use intermediaries either. Formally, the common feature of the above examples is that the traders' decisions to enter the intermediated (sub)market are non-monotone in type.<sup>2</sup> The surprising

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<sup>1</sup>But note that very different outcomes can occur in similar markets: for example, on 2 December 2009, the most expensive family residence in London available for sale online had an asking price of \$66 million. The most expensive one in Rome, a market with not too dissimilar supply and demand, was on offer at \$8.7 million, a figure certainly well below the prices paid for residences at upper end of the Roman market. This dichotomy is neatly captured by the multiplicity of equilibria in our model.

<sup>2</sup>Solid empirical evidence linking types and propensity to use intermediaries is hard to come by. The recent paper by Hitsch et al (2010), briefly discussed in Section 5, hints at some non-monotonicity. In general, however, survey studies of the intermediated marriage market

fact is not so much that the high types may coordinate on such an inefficient equilibrium – after all, examples of coordination failures abound – rather that this undesirable outcome is supported by an equilibrium at all. In this article we investigate the features of a model that delivers such non-monotone entry decisions and reveals the intuition underlying these equilibria.

Given the prevalence of markets where traders have the choice between intermediated or direct trade, it is surprising that only a few of the numerous theoretical articles on intermediated markets allow direct and intermediated trade to coexist. Gehrig (1993) is the seminal paper in this area: he posits a one-shot random matching market, where the maximisation of overall gains from trade requires matching high valuation buyers with low valuation sellers: efficient matching is negatively assortative. In his equilibrium, buyers and sellers who trade are separated by a “threshold”: buyers (sellers) with valuation below (above) the threshold trade in the direct market, buyers (sellers) with valuation above (below) the threshold trade via the intermediary. Gehrig’s finding that trade is via the intermediary for the matches with the highest surplus is confirmed by Fingleton (1997) who models intermediaries as suppliers of liquidity, in the sense that they buy from the sellers before securing a buyer to sell to. Rather than a flat fee, in Yavaş (1994) the intermediary charges a commission on the gains from trade. A second difference is that Yavaş (1994) is a search, not a matching model. Both modelling assumptions make intermediation the less attractive the higher a type is and thus, quite naturally, he finds a reverse threshold: traders’ participation strategies are monotone in type, but the keener types search directly and the less keen go to the intermediary.

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(e.g. Goodwin (1990), Bozon and Heran (1989), Kalmijn and Flap (2001) and Rosenfeld and Thomas (2010)), the well established literature on users of real estate agents (Zumpano et al 1996), and the more limited one on job exchanges (Gregg and Wadsworth 1996) contain very limited information about users’ types. Conversely, comprehensive studies of individual preferences in marriage markets, such as Choo and Siow (2006), do not have information about use of intermediation.

A crucial feature of markets with optional intermediation is that each trader’s willingness to employ the intermediary depends on which traders decide to join on the other side of the market. The “coordination game” nature of this situation naturally leads to multiple equilibria. In the existing literature, as mentioned above, all equilibria are similar in nature, characterised by a single threshold, and hence do not tally with the empirical regularities motivating our paper. We show here that to explain these observations, it is necessary to eschew the simplified modelling of negotiation between matched traders used by the previous literature, which either studied static models (as discussed above) or assumed a dynamic set-up, but with non-transferable utility (Bloch and Ryder, 2000). In contrast, our model displays a richer dynamic set-up where, crucially, a matched trader’s share of the surplus depends on the continuation payoffs, and therefore on which equilibrium the market finds itself in.

We consider a two-sided market where the traders’ types are complements: higher types benefit more from trade with higher types than lower types do. There are two trading periods, and in the first one each trader chooses whether or not to pay a fixed fee to “join” the intermediary. If they join, they are matched assortatively among those who have *joined the club*; if they stay out, they are randomly matched among those who have stayed out. Once matched, traders negotiate; crucially, refusing an exchange does not entail the loss of all benefit from trade because all the agents who have not traded can re-enter the market later.<sup>3</sup> For convenience, we collapse the future into a single second period. In this dynamic set-up, the outcome of bargaining is a function of the continuation values of the traders, which in turn depend on their type. We find equilibria with the following features: high quality traders trade in the direct search market, and may suffer a delay in finding a suitable partner;

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<sup>3</sup>Burani (2008) has a similar, fully dynamic set-up; however, for tractability, she needs to restrict attention to two types on each side of the market, ruling out non-monotone equilibria by construction.

traders with medium quality use the intermediary, and so trade immediately with probability 1; low quality traders also search directly, even when the price of intermediation is *per se* insufficient to deter them. The middle quality agents who use the intermediary are therefore “sandwiched” between the high and low quality ones who do not, and so we label this a “sandwich equilibrium”.

We show that, for a sandwich equilibrium to exist, the traders’ bargaining share must be sufficiently increasing in their own type.<sup>4</sup> The logic of this requirement can be gleaned by considering the decisions of two types: a high type,  $H$ , who does not join the intermediary, and a medium type,  $M$ , who does join. In order for  $H$  not to want to deviate from her equilibrium strategy and join the intermediary instead, the probability of meeting a low type in the open market must be sufficiently low. Since matching is type-independent, this is true for every trader. That is, any agent who stays out must have a relatively high chance of meeting a high type. Given this, why does  $M$  not want to deviate? What stops staying out and perhaps meeting a high type from becoming an alluring prospect for  $M$ ? It must be that if  $M$  meets  $H$ , he receives a low enough share of the surplus. Given the Nash bargaining protocol, this in turn is possible if  $M$  has a sufficiently lower continuation value than  $H$ .

The plan of the paper is as follows. The model is presented in Section 2, and the results in Section 3. There, in Proposition 3, we show that sandwich equilibria do indeed exist if the continuation value is increasing in type and in Proposition 4 that they do not if the continuation value is the same for all traders. Section 4 discusses the results and reports some numerical simulations, which indicates that the set of sandwich equilibria is “large”. Section 5 concludes.

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<sup>4</sup>In particular, with non-transferable utility, as assumed by Bloch and Ryder (2000), sandwich equilibria cannot happen.

## 2 The model

### 2.1 The traders

We study a two-sided market where the participants meet in pairs and share the surplus jointly available to them. For specificity we refer to the two sides as buyers and sellers, but the set-up clearly applies to more general situations, such as the marriage market. Each trader is characterised by an attribute: a seller by the quality she offers, denoted by  $q$ , and a buyer by the value he places on quality,  $v$ .

There is the same (large) number of risk neutral traders on each side of the market. We assume that the buyers' values and the sellers' qualities are drawn from strictly increasing distributions  $F_B(v)$  and  $F_S(q)$ . For convenience, we choose the measurement of qualities and values in such a way that they are all in  $[0, 1]$ , and that  $F_B(t) = F_S(t)$  for every  $t \in [0, 1]$ . Therefore, we also refer to a generic trader's type as  $t \in [0, 1]$ , with values drawn from the strictly increasing distribution  $F(t)$ ; its density is denoted by  $f(t)$ . We will be studying the limit situation as the number of traders tends to infinity, so that the law of large numbers applies.

We assume complementarity between the attributes of traders: a high value buyer appreciates an increase in quality more than a low value buyer. Note that this means that the most efficient matching is the perfectly assortative one (see Becker, 1973). For definiteness, we assume that the joint gross surplus available to the matched pair of a seller of quality  $q$  and a buyer of value  $v$  is given by  $2vq$ .

Within a matched pair of traders, utility is transferable and the outcome of negotiation is given by the Nash bargaining solution. That is, each trader obtains the sum of (i) the payoff he/she would obtain if trade did not happen – the status quo (or continuation) payoff –, and (ii) one half of the net surplus from trade. The latter is given by the difference between the gross surplus and



the sum of the parties' status quo payoffs. If it is negative, no trade takes place.

The nature of the status quo payoff, whether it is exogenously given or endogenously determined, has a crucial influence on the equilibrium set. In a static set-up, there is no future and so the status quo payoff is exogenously fixed at 0. As we show below, in this case it is impossible to capture the empirical regularities mentioned in the Introduction. We therefore posit a dynamic set-up, and deliberately choose an extremely simple one.

There are two periods. At the beginning of the first period, traders on both sides of the market simultaneously choose whether they wish to participate in the direct market or to employ the intermediary. Their choices create two submarkets: the intermediated and the direct market. Within each submarket all the traders on the short side get matched (with equal probability) to someone on the long side. That is, we assume that the matching technology is efficient. If the matched traders trade, they leave the market. If they do not trade, they stay in the market, and will be matched again in the second period. Any unmatched trader also stays in the market. There is no entry of new traders. The continuation payoff of the traders who return to the market satisfies the following condition.

**Assumption 1** *The present value of trading in the second period equals a proportion  $\lambda \in (0, 1)$  of the utility a trader would receive if in the second period he/she were perfectly assortatively matched among the remaining traders.*

$\lambda$  is a measure of the cost of delaying trade. For example, if, in the second period, all traders who have not traded in the first period were indeed matched assortatively, say by a free intermediary, then  $\lambda$  would simply be the discount factor. Alternatively, Assumption 1 holds if traders use hyperbolic discounting, whereby the discounted present value of a reward  $W$  in period  $t > 0$  is given by  $\lambda\gamma^t W$ , with  $\gamma \cong 1$ . This means that traders believe that they will become infinitely patient from the next period onwards, and therefore believe that

they will be willing to wait indefinitely for the perfect match, leading to the efficient assortative matching.<sup>5</sup> Assumption 1 ensures that, in equilibrium, a trader’s continuation payoff is a fixed proportion of the surplus from trade with a partner of matching type. This is a convenient – but clearly not the only – way to capture the requirement that the continuation payoff of a trader increases with his/her type. This will be seen to be crucial to establish the existence of equilibria with the ability to capture the empirical regularities which motivate our paper.

## 2.2 The intermediary

The supermodular nature of the game we study implies that the Pareto efficient outcome is perfect assortative matching. If traders are matched randomly in the open market, the efficiency of the matching can be increased by an intermediary. In our set-up, the intermediary reduces the mismatch cost, whereby, in order to avoid future search costs, an agent may accept to trade even though she knows that there are better partners available.

The simplest form of intermediation is one where the intermediary selects as “members”, through price or other means, only a subset of traders from both sides of the market (see, for example, Damiano and Li, 2007), and subsequently matches its members randomly. This merely ensures that members meet with members only, but already reduces mismatch. In practice, intermediaries, from tour operators through online dating services to wholesalers of grain and tea,<sup>6</sup>

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<sup>5</sup>Note that our set-up satisfies the sufficient conditions proposed by Shimer and Smith (2000) for assortative matching in a dynamic search equilibrium. Damiano et al. (2005) imply that such a dynamic matching model would unravel and lead to pooling instead of assortative matching. However, their result depends crucially on the assumption that there are per period fixed costs of participation. With discounting only, as in our model, assortative matching is the limit for infinitely patient players. Lu and McAfee (1996) use simulations to establish this in a closely related model (without intermediation).

<sup>6</sup>See for example, Clerides et al. (2008), Hitsch et al (2010) Gabre-Mahdin (2001) and

make very good use of their understanding of their members' characteristics to improve the efficiency of matches, by arranging the traders into fine clusters and restricting matches to within each cluster. This practice approximates assortative matching, and, for the sake of simplicity, we take it to the extreme and assume that the intermediary has access to a costless, perfectly assortative matching technology. Thus the intermediary will match the highest members from each side of the markets, the two second highest ones, and so on. The low ranked members on the long side will remain unmatched, and try their luck in the second period. Some related analysis shows that perfect assortative matching is not a particularly strong assumption: McAfee (2002) and Hoppe et al (2008) show that the loss of efficiency from a "coarse" relative to a "fine" clustering is very low.

Joining<sup>7</sup> the intermediary entails paying an exogenously given joining fee  $c \geq 0$ , which is the same for all traders.<sup>8</sup>

We end this Section with a summary of the extensive form of the game.

1. *At the beginning of Period 1 all traders simultaneously decide whether to join the intermediary and pay the joining fee  $c \geq 0$ , or to trade in the direct market.*
2. *Matching takes place in the two separate submarkets. If more traders join the intermediary from one side, then the traders on this side whose ranking is lower than the lowest ranking on the other side remain unmatched.*

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Koo and Lo (2004), respectively. Biglaiser (1993) gives a theoretical treatment.

<sup>7</sup>As will become apparent, our model is equivalent to a situation where the intermediary's fee is paid only by the traders who are matched.

<sup>8</sup>Notice how our model could be reinterpreted as a special case of two-sided markets, where traders are restricted to trade via "platforms". One platform is direct trade, the other is the intermediary. The former has lower efficiency and an exogenously given zero fee. To our knowledge, such a set-up has not been analysed in this literature. The most closely related papers are Armstrong (2006), Caillaud and Jullien (2003), Damiano and Li (2007, 2008) and Rochet and Tirole (2003).

*Within each matched pair, voluntary trade occurs, according to the Nash bargaining solution. Buyers and sellers who trade leave the game.*

3. *In Period 2, all remaining traders are assortatively matched and they trade, according to the Nash bargaining solution (with zero status quo payoffs). Seen from Period 1, the payoffs from agreements in Period 2 are discounted by  $\lambda$ .*

### 3 Equilibrium analysis

We begin by calculating the payoffs in the case where the joining decisions mirror each other on the two sides of the market. Let  $A$  be the set of types that do not join the intermediary, and let  $\mu(A)$  denote the measure of buyers (and sellers) who do not join.

**Proposition 1** *If the set of types that join the intermediary is the same on both sides, then the payoff of a trader of type  $t \in [0, 1]$  is*

$$t^2 - c, \tag{1}$$

*if he/she joins the intermediary, and is arbitrarily close<sup>9</sup> to*

$$\lambda t^2 + \frac{\int_A \max \left\{ tq - \frac{\lambda}{2}(q^2 + t^2), 0 \right\} f(q) dq}{\mu(A)}, \tag{2}$$

*if he/she does not join.*

**Proof.** (1) is obvious: since the intermediary matches the members assortatively, and since the  $n$ -th ranked type that joins the intermediary is the same on both sides of the market, type  $t$  is matched with type  $t$  and they trade immediately. By doing so, they obtain payoff  $t^2 - c$ , because, since they have the same outside option, they share

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<sup>9</sup>To avoid repetition in the rest of the paper the qualifier “arbitrarily close” ( $x$  and  $y$  are arbitrarily close if, for any  $\varepsilon > 0$ , there exists  $n_\varepsilon$  such that if the number of traders exceeds  $n_\varepsilon$ , then  $|x - y| < \varepsilon$ ) is kept implicit.

the gross surplus equally, from which the joining fee is subtracted. Consider next (2), the payoff for not joining the intermediary. This is simply the weighted average payoff of all possible matches. The probability of a match with type  $q$  is approximated, for  $n$  going to infinity, by  $\frac{f(q)}{\mu(A)}$ . The payoff to type  $t$  following a match with type  $q$  is  $\max\{tq - \frac{\lambda}{2}(q^2 + t^2), 0\}$ . To show this, note that in equilibrium, the continuation payoff of a type  $t$  trader is (arbitrarily close to)  $\lambda t^2$ . This is the case because, in period 2, the type distribution is the same on the two sides of the market: in the first period symmetry holds by assumption and, by the law of large numbers, the distribution of “leavers” is also the same on both sides.<sup>10</sup> Therefore, trade between  $t$  and  $q$  occurs in the first period if and only if they obtain a non-negative surplus from trading, that is if  $2tq - \lambda(q^2 + t^2) \geq 0$ , and (2) follows. ■

Based on the proof of Proposition 1, Corollary 1 identifies which matches lead to trade.

**Corollary 1** *If the set of types that join the intermediary is the same on both sides, a type  $v$  trader trades with a type  $q$  trader in the first period if and only if  $q \in \left[\frac{1-\sqrt{1-\lambda^2}}{\lambda}v, \frac{1+\sqrt{1-\lambda^2}}{\lambda}v\right]$ .*

Figure 1 illustrates this: trade occurs only if the matched traders’ type vector is in the grey area. Notice that, since  $\lambda \rightarrow 1$  implies  $\frac{1-\sqrt{1-\lambda^2}}{\lambda} \rightarrow 1$ , as  $\lambda$  increases, the grey area shrinks to the diagonal: if traders are infinitely patient, they are unwilling to “trade down” and matching must be assortative. Vice versa, if  $\lambda \rightarrow 0$ , we have  $\frac{1-\sqrt{1-\lambda^2}}{\lambda} \rightarrow 0$ , and the grey area tends to the whole square  $[0, 1]^2$ : if waiting becomes infinitely costly then any match leads to trade as the gross surplus is non-negative.

As mentioned in the Introduction, the equilibria derived in the literature stratify the types into two groups. Definition 1 captures this idea.

<sup>10</sup>Formally, let  $A \subseteq [0, 1]$  be the set of buyers and sellers in the market in period 2. Then, for every  $\varepsilon > 0$ , there exists a number of traders  $n$  high enough such that, if  $q_b^n(v, x)$  is the proportion of type  $v$  buyers who meet a type  $x$  seller, and  $q_s^n(v, x)$  is the proportion of type  $v$  sellers who meet a type  $x$  buyer, then  $\max_{(v,x) \in A \times A} |q_b^n(v, x) - q_s^n(v, x)| < \varepsilon$ .

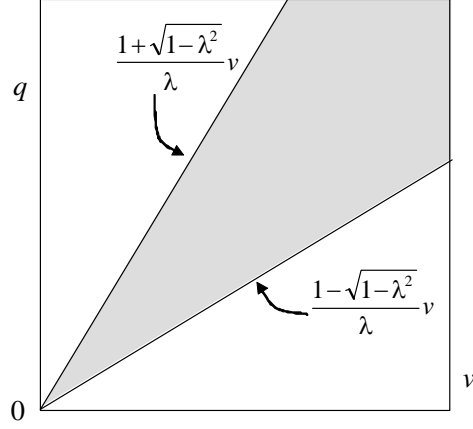


Figure 1: The set of matched pairs who agree to trade in period 1.

**Definition 1** *A  $t$ -threshold equilibrium is a SPE with the property that all traders with type greater than or equal to  $t$  join the intermediary.*

As one would expect, our model exhibits a rich multiplicity of threshold equilibria, as the following proposition shows. Let  $r = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$  and  $\sigma(\lambda) = \lambda + \int_r^1 (q - \frac{\lambda}{2}(q^2 + 1)) f(q) dq$ . Note that  $\sigma(\lambda) \in [0, 1]$ , with  $\sigma(1) = 1$ .

**Proposition 2** *For every joining fee  $c \in [0, 1 - \sigma(\lambda)]$ , there exists  $x(c) \in [0, 1)$  such that a  $t$ -threshold equilibrium exists for every  $t \in [x(c), 1]$ .*

**Proof.** Fix a putative  $t$ -threshold equilibrium. By Proposition 1, types  $v \geq t$  expect a payoff of  $v^2 - c$  in equilibrium, whereas deviating implies an expected payoff of  $\lambda v^2 + \frac{\int_0^t \max\{vq - \frac{\lambda}{2}(q^2 + v^2), 0\} f(q) dq}{F(t)}$  which, by Corollary 1, equals  $\lambda v^2 + \frac{\int_{vr}^t (vq - \frac{\lambda}{2}(q^2 + v^2)) f(q) dq}{F(t)}$ . Let  $D(t, v)$  be the difference between the equilibrium and deviation payoffs for type  $v \geq t$ . We start with establishing that  $D(t, v)$  is increasing in  $v$ , so that it is sufficient to check incentive compatibility for the threshold type,  $v = t$ .

$$\begin{aligned} \frac{\partial D(t, v)}{\partial v} &= 2v(1 - \lambda) - \int_{vr}^t \frac{(q - \lambda v) f(q)}{F(t)} dq + \frac{r(v^2 r - \frac{\lambda}{2}(r^2 v^2 + v^2)) f(vr)}{F(t)} \\ &\geq 2t(1 - \lambda) - \int_{vr}^t \frac{(t - \lambda t) f(q)}{F(t)} dq = t(1 - \lambda) \left(1 + \frac{F(vr)}{F(t)}\right) \geq 0. \end{aligned}$$

Here the first inequality follows from the fact that  $v \geq t \geq q$  and that the last term is 0, as the lower bound of the integral is by definition the value of  $q$  for which the integrand is 0.

Next, observe that  $D(1, 1) = 1 - c - \sigma(\lambda)$ . As the integrand in  $\sigma(\lambda)$  is strictly increasing, we obtain a strict upper bound on  $\sigma(\lambda)$  by setting  $q = 1$  in the integrand:  $\sigma(\lambda) < \lambda + (1 - \lambda)(1 - F(r)) < 1$ . Hence,  $1 - \sigma(\lambda) > 0$ , so for  $c \in [0, 1 - \sigma(\lambda))$ ,  $D(1, 1) > 0$ .

Then, by the continuity of the function  $D(t, t)$  in  $t$ , for any  $c \in [0, 1 - \sigma(\lambda))$  there exists a type  $x(c) < 1$ , such that for all  $v > x(c)$ , we have  $D(v, v) > 0$ .

Finally, consider types  $v < t$ . Their equilibrium payoff is at least  $\lambda v^2$ , while their deviation payoff is  $\lambda v^2 - c$ , since by Summary 2 they would not find a match with the intermediary. Hence deviation is not profitable. ■

When intermediation is free, a more precise characterisation is possible:  $x(0) = 0$ .

**Corollary 2** *When  $c = 0$  there is a threshold equilibrium for every  $t \in [0, 1]$ .*

**Proof.** Take any  $t \geq 0$ . We show that there exists a  $t$ -threshold equilibrium, which establishes the result. Consider first types below  $t$ : in the second period they are assortatively matched, and so their period 1 reservation payoff is the same in equilibrium and following a deviation. However they trade with zero probability in period 1 if they deviate (the intermediary will not match them), and with non-zero probability in equilibrium. Therefore following their putative equilibrium strategy, staying out, is preferable. Consider now a type  $v \geq t$ . If she follows the equilibrium strategy, she is matched by the intermediary to type  $v$ , trades, and obtains a payoff of  $v^2$ . If she deviates, she is matched to type  $q < t$ , which gives her a payoff of  $\lambda v^2$  if she does not trade and  $\lambda v^2 + vq - \frac{\lambda}{2}(v^2 + q^2)$  if she trades. Since the last expression is strictly increasing in  $q$  for  $q < v$  and equals  $v^2$  when  $q = v$ , it is no more than  $v^2$  for  $q \leq v$ , and so type  $v$  does not gain by deviating from her putative equilibrium strategy. This shows that no-one has an incentive to deviate and so there is a  $t$ -threshold equilibrium. ■

That is, if joining the intermediary is free, then any type  $t \in [0, 1]$  can be the threshold in a  $t$ -threshold equilibrium. A further consequence is that, for any  $\lambda \in [0, 1]$ , there exists a 0-threshold equilibrium if and only if  $c = 0$ . In words, with no intermediation fee, and only with no intermediation fee, it is an equilibrium for all traders to join, which is the efficient outcome, given our assumption that the intermediary can sort costlessly.

Threshold equilibria are identified in the existing literature, and have a natural explanation: the top traders join a club, which, although open to all who are willing to pay the fee, has little use for those whose valuation and quality is below the threshold, as they will be cold-shouldered by the members.

But this natural equilibrium configuration is not the only possible one. We show next that there are equilibria where only traders with intermediate types join the intermediary, while “top” and “bottom” type traders search directly.

**Definition 2** A  $\{\underline{t}, \bar{t}\}$ -sandwich equilibrium with  $0 \leq \underline{t} < \bar{t} < 1$  is a SPE, where in period 1 a trader joins the intermediary if and only if he/she has type  $t \in [\underline{t}, \bar{t}]$ . If  $0 < \underline{t}$ , the sandwich equilibrium is non-degenerate.

The following lemma, which we need in the proof of our main result, is of independent interest.

**Lemma 1** In a  $\{\underline{t}, \bar{t}\}$ -sandwich equilibrium, the deviation payoff of a type  $t$  trader is given by:

$$\lambda t^2 - c \quad \text{for } t \in [0, \underline{t}), \quad (3)$$

$$\lambda t \underline{t} + \frac{\int_{[0, \underline{t}) \cup (\bar{t}, 1]} \max \left\{ tq - \frac{\lambda}{2} (q^2 + t \underline{t}), 0 \right\} f(q) dq}{1 - F(\bar{t}) + F(\underline{t})} \quad \text{for } t \in [\underline{t}, \bar{t}], \quad (4)$$

$$\lambda t^2 + \max \left\{ t \bar{t} - \frac{\lambda}{2} (t^2 + \bar{t}^2), 0 \right\} - c \quad \text{for } t \in (\bar{t}, 1]. \quad (5)$$

**Proof.** If a low type ( $t \in [0, \underline{t})$ ) deviates and joins the intermediary, she pays the joining fee  $c$ , does not trade, and is assortatively matched in the next period. Her payoff is therefore  $\lambda t^2 - c$ .



Consider a middle type next:  $t \in [\underline{t}, \bar{t}]$ . Her deviation payoff is determined by the Nash bargaining solution and it is a function of her continuation value, which is  $\lambda t \underline{t}$ . To see this, note that, following a deviation, a seller of type  $q$  is either matched to a high type (with probability  $\frac{1-F(\bar{t})}{1-F(\bar{t})+F(\underline{t})}$ ) or matched to a low type (with probability  $\frac{F(\underline{t})}{1-F(\bar{t})+F(\underline{t})}$ ). In both cases, all the top buyers who reach period 2 will be matched with a seller of type equal to their own, and therefore, a deviating type  $q$  seller will be matched to the highest type who is left in the market after all the top buyers are assortatively matched, which is  $\underline{t}$ .

Finally, consider a high type buyer,  $v \in (\bar{t}, 1]$ . If he decides to join the intermediary, he will participate in an assortative matching, where he is the highest type. Consequently, he will be matched with the highest type seller who joins the intermediary in equilibrium,  $q = \bar{t}$ . This gives his deviation payoff (5), and establishes Lemma 1. ■

To evaluate (4) in Lemma 1 it is necessary to determine whether a type  $t \in [\underline{t}, \bar{t}]$ , who should join the intermediary in equilibrium, would trade in the first period, if he/she deviated instead and were matched in the direct market. To do so, compact notation by writing  $(tq - \frac{\lambda}{2}(q^2 + t^2)) f(q)$  as  $h(q, \underline{t}, t)$ , and  $\frac{1-\sqrt{1-\lambda^2(\underline{t}/t)}}{\lambda}$  as  $R((\underline{t}/t))$ , and write (4) as:

$$\frac{\int_{tR(\underline{t}/t)}^{\underline{t}} h(q, \underline{t}, t) dq}{1 - F(\bar{t}) + F(\underline{t})} + \lambda t \underline{t} \quad \text{for } t \in \left( \underline{t}, \frac{\lambda \bar{t}^2}{2\bar{t} - \lambda \underline{t}} \right), \quad (6)$$

$$\frac{\int_{tR(\underline{t}/t)}^{\underline{t}} h(q, \underline{t}, t) dq}{1 - F(\bar{t}) + F(\underline{t})} + \frac{\int_{\bar{t}}^{\frac{\underline{t}}{R(\underline{t}/t)}} h(q, \underline{t}, t) dq}{1 - F(\bar{t}) + F(\underline{t})} + \lambda t \underline{t} \quad \text{for } t \in \left[ \frac{\lambda \bar{t}^2}{2\bar{t} - \lambda \underline{t}}, \frac{\lambda}{2 - \lambda \underline{t}} \right], \quad (7)$$

$$\frac{\int_{tR(\underline{t}/t)}^{\underline{t}} h(q, \underline{t}, t) dq}{1 - F(\bar{t}) + F(\underline{t})} + \frac{\int_{\bar{t}}^1 h(q, \underline{t}, t) dq}{1 - F(\bar{t}) + F(\underline{t})} + \lambda t \underline{t} \quad \text{for } t \in \left( \frac{\lambda}{2 - \lambda \underline{t}}, \bar{t} \right]. \quad (8)$$

Figure 2 illustrates this. The coloured subset in the diagram depicts the combinations of types  $(v, q)$  such that if a type  $v \in [\underline{t}, \bar{t}]$  deviates and is matched to type  $q \in [0, \underline{t}) \cup (\bar{t}, 1]$ , then trade occurs. These points are those above and to the right of the locus determined by the two curves  $q = \frac{v - \sqrt{v^2 - \lambda^2 v \underline{t}}}{\lambda}$  and  $q = \frac{v + \sqrt{v^2 - \lambda^2 v \underline{t}}}{\lambda}$ . This locus intersects the line  $\underline{t}$  at  $v = \frac{\lambda \underline{t}}{2 - \lambda}$ ; it also intersects the

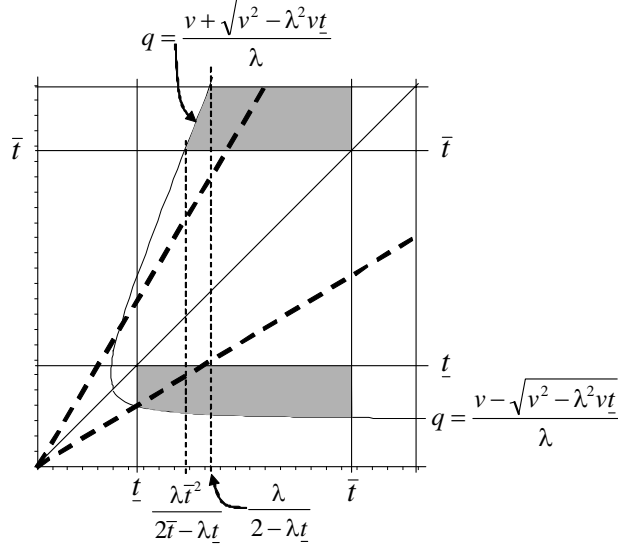


Figure 2: Trade following a deviation by a type  $t \in [\underline{t}, \bar{t}]$ .

diagonal at  $v = \frac{\lambda \underline{t}}{2 - \lambda}$ . It reaches its leftmost point (a minimum on the vertical axis) at point  $(v = \lambda^2 \underline{t}, q = \lambda \underline{t})$ , and it has a horizontal asymptote at  $q = \frac{\lambda}{2} \underline{t}$ . Finally, it intersect the line  $q = 1$  at  $v = \frac{\lambda}{2 - \lambda \underline{t}}$ , and the line  $\frac{v}{\lambda} (1 - \sqrt{1 - \lambda^2})$  at  $v = \underline{t}$ .

We are now ready to present our main result.

**Proposition 3** *If the density function  $f(t)$  is Lipschitz continuous at 0, then for any  $\lambda > \frac{1}{2}$ , there exists  $c^* > 0$  such that, for every  $c \in [0, c^*]$ , there exists a non-degenerate  $\{\underline{t}, \bar{t}\}$ -sandwich equilibrium.*

**Proof.** We begin by considering the special case of a degenerate sandwich equilibrium,  $\underline{t} = 0$ , no entry fee, and  $c = 0$ . Our argument will be based on the concept of  $\{\underline{t}, \bar{t}\}$ -almost strict sandwich equilibrium, ( $\{\underline{t}, \bar{t}\}$ -ASSE). This is a  $\{\underline{t}, \bar{t}\}$ -sandwich equilibrium where all traders, with the exception of trader  $t = 0$ , *strictly* prefer to follow their equilibrium strategy rather than deviate; trader  $t = 0$  is indifferent. We first show that there are  $\{0, \bar{t}\}$ -ASSE's. Next we establish the existence of a  $\{\varepsilon, \bar{t}\}$ -ASSE: it is possible to increase the lower bound of the set of “joiners” slightly above 0, to some  $\varepsilon > 0$ , and maintain the property that all traders, including traders in

$(0, \varepsilon]$ , strictly prefer to follow their equilibrium strategy rather than to deviate. This establishes the proposition for the special case  $c = 0$ . Since all traders who join the intermediary strictly prefer to do so, it is possible to choose a positive  $c$  such that this continues to be the case, establishing the Proposition.

To begin, therefore, we first want to show that, for some  $\bar{t} \in (0, 1)$ , when  $\underline{t} = 0$  and  $c = 0$ , the types in  $(0, \bar{t})$  (the “degenerate” middle) prefer to join the intermediary and the types above  $\bar{t}$  prefer not to. Formally:

$$t^2 > \frac{\int_{(\bar{t}, 1]} \max \left\{ tq - \frac{\lambda}{2} q^2, 0 \right\} f(q) dq}{1 - F(\bar{t})} \quad \text{for } t \in (0, \bar{t}], \quad (9)$$

$$\frac{\int_{(\bar{t}, 1]} \max \left\{ tq - \frac{\lambda}{2} (q^2 + t^2), 0 \right\} f(q) dq}{1 - F(\bar{t})} > \max \left\{ t\bar{t} - \frac{\lambda}{2} (t^2 + \bar{t}^2), 0 \right\} \quad \text{for } t \in [\bar{t}, 1]. \quad (10)$$

To ensure almost strictness, we require that both (9) and (10) be satisfied at  $\bar{t}$ . Take constraint (10), and evaluate it for the marginal type,  $t = \bar{t}$ . We need to show that

$$\frac{\int_{(\bar{t}, 1]} \max \left\{ \bar{t}q - \frac{\lambda}{2} (q^2 + \bar{t}^2), 0 \right\} f(q) dq}{1 - F(\bar{t})} > (1 - \lambda) \bar{t}^2.$$

This is implied by

$$\frac{\int_{(\bar{t}, 1]} (\bar{t}q - \frac{\lambda}{2} q^2) f(q) dq}{1 - F(\bar{t})} > \left(1 - \frac{\lambda}{2}\right) \bar{t}^2. \quad (11)$$

Note that the LHS in (11) is the average value of the function  $g(q) = \bar{t}q - \frac{\lambda}{2} q^2$  in the interval  $[\bar{t}, 1]$ , while the RHS is  $g(\bar{t})$ .  $g(q)$  is a negative quadratic and hence it reaches its minimum on  $[\bar{t}, 1]$  either at  $\bar{t}$  or at 1. Consequently, a sufficient condition for the strict inequality to hold is that  $g(\bar{t}) = \bar{t}^2 (1 - \frac{\lambda}{2}) \leq \bar{t} - \frac{\lambda}{2} = g(1)$ . This is equivalent to  $\bar{t} \in \left[\frac{\lambda}{2-\lambda}, 1\right]$ , which is non-empty for all  $\lambda \in (0, 1)$ . Next, we show that if the inequality is satisfied for the marginal type,  $t = \bar{t}$ , it is also satisfied for all types  $t > \bar{t}$ . Note first that whenever  $tq - \frac{\lambda}{2} (q^2 + t^2) > t\bar{t} - \frac{\lambda}{2} (t^2 + \bar{t}^2)$  for every  $t$ , it is also true that  $q \in [\bar{t}, 1]$ , and therefore (10) is implied by

$$\frac{\int_{(\bar{t}, 1]} (tq - \frac{\lambda}{2} (q^2 + t^2)) f(q) dq}{1 - F(\bar{t})} - t\bar{t} + \frac{\lambda}{2} (t^2 + \bar{t}^2) > 0 \quad \text{for } t \in [\bar{t}, 1].$$

Differentiate the LHS of the above with respect to  $t$ :

$$\frac{\int_{[\bar{t}, 1]} (q - \lambda t) f(q) dq}{1 - F(\bar{t})} - (\bar{t} - \lambda t).$$

Note that the first term is the average of  $h(q) = q - \lambda t$  in the interval  $[\bar{t}, 1]$ , while the second is  $h(\bar{t})$ . As  $h(q)$  is increasing in  $q$ , the above is positive and so (10) holds for  $\bar{t} \in \left[\frac{\lambda}{2-\lambda}, 1\right]$ .

Next consider (9), which requires that types in  $[0, \bar{t}]$  prefer to join the intermediary rather than deviate. (9) can be written as expressions (6)-(8), which, for  $t = 0$ , reduce to (note that  $\lim_{t \rightarrow 0} R(\underline{t}/t) = \frac{2}{\lambda}t$ ):

$$\begin{aligned} t^2 &> 0 & \text{for } t &\in \left(0, \frac{\lambda \bar{t}}{2}\right), \\ t^2 &> \frac{\int_{\bar{t}}^{\frac{t}{\lambda}} (tq - \frac{\lambda}{2}q^2) f(q) dq}{1 - F(\bar{t})} & \text{for } t &\in \left[\frac{\lambda \bar{t}}{2}, \frac{\lambda}{2}\right], \\ t^2 &> \frac{\int_{\bar{t}}^1 (tq - \frac{\lambda}{2}q^2) f(q) dq}{1 - F(\bar{t})} & \text{for } t &\in \left(\frac{\lambda}{2}, \bar{t}\right]. \end{aligned}$$

The first line is clearly true. Consider the second. Since  $\frac{2}{\lambda}t < 1$ , the RHS does not exceed the average of  $k(q) = tq - \frac{\lambda}{2}q^2$  in the set  $\left[\bar{t}, \frac{2\bar{t}}{\lambda}\right]$ .  $k(q)$  is a negative quadratic with its global maximum at  $q = \frac{\bar{t}}{\lambda}$ , which is to the left of  $\frac{2\bar{t}}{\lambda}$ . Therefore its maximum in  $\left[\bar{t}, \frac{2\bar{t}}{\lambda}\right]$  is either at  $q = \bar{t}$  or at  $q = \frac{\bar{t}}{\lambda}$ . Hence, a sufficient condition for the second line to hold with slack (given that  $k(q)$  is not constant) is  $t^2 \geq \max\left\{k(\bar{t}), k\left(\frac{\bar{t}}{\lambda}\right)\right\} = \max\left\{t\bar{t} - \frac{\lambda}{2}\bar{t}^2, \frac{\bar{t}^2}{2\lambda}\right\}$ . When  $t \leq \bar{t}$  the LHS of the quadratic inequality  $t^2 - t\bar{t} + \frac{\lambda}{2}\bar{t}^2 \geq 0$  has no real roots for  $\lambda \geq \frac{1}{2}$ . Therefore for  $\lambda \geq \frac{1}{2}$  the condition in the second line is always satisfied.

Finally, the third line. We have the same situation as with the second line, except that now the maximum of  $k(q)$  might be reached at  $q = 1$ , as  $\frac{t}{\lambda}$  may be greater than one. Hence, we have the additional condition:  $t^2 \geq t - \frac{\lambda}{2}$ , which again is guaranteed for  $\lambda \geq \frac{1}{2}$ .

We have thus shown that when  $\lambda \geq \frac{1}{2}$  and  $c = 0$ , there exists a  $\{0, \bar{t}\}$ -ASSE for any  $\bar{t} \geq \frac{\lambda}{2-\lambda} \geq \frac{1}{3}$ . The next Lemma ensures that this continues to be the case if the lower bound of the middle group, those who join the intermediary, is increased slightly.

**Lemma 2** *Let  $c = 0$ , and let  $\bar{t}$  be such that there exists a  $\{0, \bar{t}\}$ -ASSE. Then there exists  $\varepsilon > 0$  such that there exists an  $\{\varepsilon, \bar{t}\}$ -ASSE.*

**Proof.** Take a  $\{0, \bar{t}\}$ -ASSE and a  $\underline{t} > 0$ . All types  $t \in (0, \underline{t})$  will still strictly prefer not to deviate, as by symmetry and Summary 2, they will not be matched if they join the intermediary. Types in  $(\bar{t}, 1]$ , will become more inclined to deviate as they now might be matched with a trader from the bottom slice. However, as we have started from a  $\{0, \bar{t}\}$ -ASSE, by the continuity of payoffs in  $\underline{t}$  (due to the Lipschitz continuity of  $f$ ), we can take a small enough  $\underline{t} > 0$  that keeps the top types from deviating. Thus, to establish the lemma we only need to show that for some (small)  $\underline{t} > 0$ , types  $t \in [\underline{t}, \bar{t}]$  strictly prefer not to deviate. Since we had a  $\{0, \bar{t}\}$ -ASSE, (7) and (8) are still satisfied for  $\underline{t} > 0$  sufficiently small, so we only need to check that (6) holds. Rewriting it in full:

$$t^2 - \frac{\int_{\underline{t} \frac{1 - \sqrt{1 - \lambda^2 \underline{t}}}{\lambda}}^{\underline{t}} (tq - \frac{\lambda}{2} (q^2 + \underline{t}t)) f(q) dq}{1 - F(\bar{t}) + F(\underline{t})} - \lambda \underline{t} t > 0 \quad \text{for } t \in \left( \underline{t}, \frac{\lambda \bar{t}^2}{2\underline{t} - \lambda \underline{t}} \right). \quad (12)$$

We first evaluate (12) at  $t = \underline{t}$ , which is the lower end of the range, obtaining,

$$(1 - \lambda)t^2 - \frac{1}{P(t)} \int_{tr}^t \left( tq - \frac{\lambda}{2} (t^2 + q^2) \right) f(q) dq > 0, \quad (13)$$

since  $t > 0$ . Recall that  $r = \frac{1 - \sqrt{1 - \lambda^2}}{\lambda}$ , and let  $P(t) = 1 - F(\bar{t}) + F(t)$ . Next differentiate the LHS of (13) with respect to  $t$ :

$$2(1 - \lambda)t + \frac{f(t)}{P(t)^2} \int_{tr}^t \left( tq - \frac{\lambda}{2} (t^2 + q^2) \right) f(q) dq + \frac{1}{P(t)} \left\{ (1 - \lambda)t^2 f(t) - r \left( r - \frac{\lambda}{2} (1 + r^2) \right) t^2 f(rt) + \int_{tr}^t (q - \lambda t) f(q) dq \right\}.$$

Note that  $r - \frac{\lambda}{2} (1 + r^2) = 0$ , and so the above is

$$2(1 - \lambda)t + \frac{f(t)}{P(t)^2} \int_{tr}^t \left( tq - \frac{\lambda}{2} (t^2 + q^2) \right) f(q) dq + \frac{1}{P(t)} \left\{ (1 - \lambda)t^2 f(t) + \int_{tr}^t (q - \lambda t) f(q) dq \right\},$$

which is 0 at  $t = 0$ . Had this been positive the proof would be complete. Instead, we need to check the second derivative:

$$2(1-\lambda) + \frac{d\frac{f(t)}{P(t)^2}}{dt} \int_{tr}^t \left( tq - \frac{\lambda}{2}(t^2 + q^2) \right) f(q) dq +$$

$$\frac{2f(t)}{P(t)^2} \left\{ (1-\lambda)t^2 f(t) + \int_{tr}^t (q - \lambda t) f(q) dq \right\} -$$

$$\frac{1}{P(t)} \left\{ 2(1-\lambda)tf(t) + (1-\lambda)t^2 f'(t) + t(1-\lambda)f(t) - r(tr - \lambda t)f(tr) - \int_{tr}^t \lambda f(q) dq \right\}.$$

Evaluating the above at  $t = 0$ :

$$2(1-\lambda) + \frac{d\frac{f(t)}{P(t)^2}}{dt} \int_{tr}^t \left( tq - \frac{\lambda}{2}(t^2 + q^2) \right) f(q) dq - \frac{(1-\lambda)t^2 f'(t)}{P(t)}.$$

or

$$2(1-\lambda) + \frac{f'(t)}{P(t)} \left\{ \frac{1}{P(t)} \int_{tr}^t \left( tq - \frac{\lambda}{2}(t^2 + q^2) \right) f(q) dq - (1-\lambda)t^2 \right\}.$$

The requirement that  $f$  be Lipschitz continuous at 0 implies that  $f'(0)$  is finite, and therefore the above is  $2(1-\lambda) > 0$  at  $t = 0$ . Hence, the LHS of (13) is convex, which establishes the Lemma. ■

The Lemma implies that, at least for a small increase in the lower bound of the interval of joiners, all the joiners strictly prefer to join the intermediary. The observation that this continues to be true for a sufficiently small increase in  $c$  establishes the main result. ■

This existence result holds regardless of the distribution,<sup>11</sup> and is therefore very general. However it clearly does not characterise fully the set of sandwich equilibria: as intuition suggests, and the technique of the proof confirms, there is a rich multiplicity of sandwich equilibria:<sup>12</sup> specifically, the set of sandwich

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<sup>11</sup>The requirement that  $f$  be Lipschitz continuous at 0 is a sufficient condition, which is used to prove existence, but is not necessary: sandwich equilibria exist even when the condition is violated.

<sup>12</sup>The defining feature of a sandwich equilibrium is that the “top” and the “bottom” traders do not join the intermediary: this feature is in contrast to the “threshold” equilibria identified by the literature, where the highest types join the intermediary. There might also

equilibria has, generically, full dimensionality in the set of possible values of  $\underline{t}$  and  $\bar{t}$ .<sup>13</sup> Intuitively, sandwich equilibria exist if a number of incentive constraints are satisfied, and the proof of Proposition 3 shows that it is possible to select  $\{\underline{t}, \bar{t}\}$  such that all these constraints are slack at the  $\{\underline{t}, \bar{t}\}$ -sandwich equilibrium. This implies that the set of sandwich equilibria contains  $\{\underline{t}, \bar{t}\}$ -sandwich equilibria which have the property that there exists  $\varepsilon$  such that for every  $\{\underline{v}, \bar{v}\}$  satisfying  $|\underline{t} - \underline{v}| + |\bar{t} - \bar{v}| < \varepsilon$  there is also a  $\{\underline{v}, \bar{v}\}$ -sandwich equilibrium. This can be interpreted as giving our equilibrium set a degree of robustness to the introduction of small errors in the matching technology available to the intermediary. Suppose, for example, that matching through the intermediary is perfectly assortative with probability  $(1 - \varepsilon)$ , and random with probability  $\varepsilon$ , with  $\varepsilon > 0$  and “small”. This would reduce slightly the benefit of joining the intermediary, given in (1), (3) and (5), and therefore make a deviation slightly less attractive for types in  $(\bar{t}, 1]$ , and slightly more attractive for types in  $[0, \underline{t})$ . Conversely, it would make following the equilibrium strategy slightly more (less) attractive for types below (above) the average of the types in  $[\underline{t}, \bar{t}]$ . Except at the boundary of the set of sandwich equilibria, a sufficiently small  $\varepsilon$  would not prevent a pair  $\{\underline{t}, \bar{t}\}$  from being a sandwich equilibrium.

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exist more complex equilibria, for example, sandwiches with three slices: the unit segment is divided in five intervals, such that type in the first, third and fifth stay out and those in the second and fourth interval join the intermediary: in this case the middle interval has a “hole”, that is there are types in  $[\underline{t}, \bar{t}]$  who do not join the intermediary. This is analogous to Bloch and Ryder’s (2000) finding that there might be threshold equilibria where there are “holes” in the distribution of joiners (Theorem 3.5, p 107). This can only happen if the intermediary charges a proportional commission, not, as here, a flat fee. We also conjecture that there exist asymmetric sandwich equilibria, where the intervals of joiners are different on the two sides of the market, though their measure is the same.

<sup>13</sup>In Section 4.5, we use computer simulations to provide a full characterisation of the set of sandwich equilibria, for specific values of the two parameters and restricting attention to the uniform distribution.

The “robust” nature of sandwich equilibria and their intuitive appeal given by their “coordination equilibrium” nature may suggest that they are somewhat easy to obtain. This, however, is definitely not the case: existence of sandwich equilibria is subject to quite stringent conditions. These conditions illustrate the crucial role played by the continuation value, which must be such that higher types have a better outside option, and therefore also provide the intuitive reason for the emergence of sandwich equilibria. The following proposition establishes this formally.<sup>14</sup>

**Proposition 4** *If the continuation value is constant across types, then there are no sandwich equilibria.*

**Proof.** Let the discounted value of the common option be denoted by  $\ell > 0$ , the same for all traders. We prove the Proposition by contradiction. Suppose therefore that there does exist a sandwich equilibrium. Let  $x$  be the supremum of types who join the intermediary at this equilibrium. Then, for any  $y > x$ , the following two inequalities must hold:

$$x^2 - c \geq xE[v|xv \geq \ell] \Pr(xv \geq \ell) + \ell \Pr(xv < \ell), \quad (14)$$

$$yE[v|yv \geq \ell] \Pr(yv \geq \ell) + \ell \Pr(yv < \ell) \geq yx - c. \quad (15)$$

The probabilities and expectations on the LHS of (14) and on the RHS of (15) are taken relative to the distribution of types who do not join in equilibrium. The LHS of both (14) and (15) is the equilibrium payoff: (14) requires that type  $x$ , who joins the intermediary and receives payoff  $x^2 - c$ , is better-off than at her outside option, otherwise she would not join. If she does not join, trade takes place if  $2yv - 2\ell \geq 0$ , splitting the gross surplus equally; otherwise she collects her outside option,  $\ell$ . The RHS in (14) and (15) is the deviation payoff: if the type who joins were to deviate, she would be randomly matched with a non-joiner and would save the fee. Vice versa, upon deviation, a type who should not join in equilibrium decided to join instead,

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<sup>14</sup>Damiano and Li (2007 p 260) conjecture that type dependent reservation utilities would not alter the “threshold” structure of the equilibria. Our paper can therefore be seen as limiting the applicability of this conjecture.



she would pay the fee and be assortatively matched with a trader on the other side, whose type will be arbitrarily close to  $x$ , the supremum of the joiners. As  $y > x$ , either trade occurs, giving  $y$  the payoff in the inequality or it is  $y$  who refuses to trade to obtain an even higher payoff.

Rearranging the inequalities we have

$$\begin{aligned} x^2 - xE[v|xv \geq \ell] \Pr(xv \geq \ell) - \ell \Pr(xv < \ell) &\geq c \\ &\geq yx - yE[v|yv \geq \ell] \Pr(yv \geq \ell) - \ell \Pr(yv < \ell). \end{aligned} \quad (16)$$

Next, note that

$$xE[v|xv \geq \ell] \Pr(xv \geq \ell) + \ell \Pr(xv < \ell) \geq xE[v|yv \geq \ell] \Pr(yv \geq \ell) + \ell \Pr(yv < \ell).$$

This holds because the LHS is type  $x$ 's optimal deviation payoff, while the RHS assumes that (following a deviation) sometimes  $x$  trades even if it gives her less than her outside option.

Hence a necessary condition for (16) to hold is that

$$\begin{aligned} x^2 - xE[v|xv \geq \ell] \Pr(yv \geq \ell) - \ell \Pr(yv < \ell) &\geq c \\ &\geq yx - yE[v|yv \geq \ell] \Pr(yv \geq \ell) - \ell \Pr(yv < \ell), \end{aligned}$$

or

$$(x - y)(x - E[v|yv \geq \ell] \Pr(yv \geq \ell)) \geq 0. \quad (17)$$

Note that, by (16),  $x - E[v|xv \geq \ell] \Pr(xv \geq \ell) \geq \frac{\ell \Pr(xv < \ell) + c}{x} > 0$ . Thus if we can show that there exists  $y > x$  such that  $E[v|yv \geq \ell] \Pr(yv \geq \ell) - E[v|xv \geq \ell] \Pr(xv \geq \ell) < \frac{\ell \Pr(xv < \ell) + c}{x}$ , then  $x - E[v|yv \geq \ell] \Pr(yv \geq \ell) > 0$  and hence (17) implies a contradiction. Such a  $y$  indeed exists because – by construction – all types between  $x$  and  $y$  do not join the intermediary; since the type distribution contains no mass points,  $E[v|yv \geq \ell] \Pr(yv \geq \ell)$  is continuous in  $y$ , so  $E[v|yv \geq \ell] \Pr(yv \geq \ell) - E[v|xv \geq \ell] \Pr(xv \geq \ell)$  converges continuously to 0 as  $y$  tends to  $x$ . ■

The logic underlying this result is that a uniform outside option has no effect on the outcome of bargaining; the gross surplus is divided equally between

the parties. This is exactly the same as when utility were non-transferable: our paper therefore shows that for sandwich equilibria to exist it is necessary that utility is non-transferable. Intuitively, the fifty-fifty arrangement implied by uniform continuation value or non-transferable utility makes meeting high types very attractive, and thus puts an upper bound on the relative probability of meeting a high type if not joining the intermediary. On the other hand, this very fifty-fifty split also makes meeting low types not very attractive, because they will have a good deal of bargaining power. To keep high types from joining the intermediary the relative probability of meeting a low type if not joining the intermediary must be low. As it turns out, these two requirements cannot be simultaneously met. To alter the constraints so as to make them compatible with each other, the bargaining outcome needs to favour the higher types. Having a continuation value which is increasing in the type does just that.

## 4 Discussion

**4.1.** We have assumed perfect information: upon meeting, the traders can observe each other's type. Asymmetric information would give rise to a potential for signalling. The decision whether or not to join the intermediary conveys some information, and thus it affects a trader's payoff not just because it affects the range of potential partners, but also because it affects the partner's beliefs about one's type. Analysis of a model which incorporates both asymmetric information and the possibility of joining an intermediary is likely to be beyond tractability. Interestingly, in a pure signalling set-up, Feltovich et al. (2002) show that there can be equilibria with non-monotone signalling strategies, also known in the literature as counter-signalling. In these equilibria, only middle types send the costly signal, while low types and high types pool by

not sending it.<sup>15</sup> The analogy with the outcome of our model suggests that, in situations where there is a binary choice (join the intermediary or send the signal), non-monotonicity of the equilibrium strategies in type is a common outcome (Renou, 2010, is another example).

**4.2.** Some of the existing literature (e.g. Yavaş, 1994, Bloch and Ryder, 2000) considers a fee proportional to the benefit from trade, as is the case when the intermediary charges a commission. This has a much higher information requirement than a flat fee, and in fact many markets (online dating, mortgage brokering and shopping centres for examples) do charge a flat fee. A second reason why we have chosen to model the case of a flat fee is that we wanted to explain the existence of sandwich equilibria, and a proportional commission makes their existence *easier*: with a flat fee, when middle types join, it is certainly the case that the high types will not forgo intermediation due to its cost, as it may happen with a proportional commission. Finally, note that the existence of non-degenerate sandwich equilibria is not due to the flat fee excluding the lower quality segment from intermediation, since our result holds for zero fee.

**4.3.** A further assumption we made is that the intermediary has access to a perfect matching technology, whereas trading in the direct market gives rise to perfectly random matching. In practice, of course, both are approximations. Relaxing them has in general an ambiguous effect on the attractiveness of using the intermediary. On the one hand, real world trading in the direct market is far from random: for example, high quality singles who do not join a lonely

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<sup>15</sup>As an example, suppose there are three types,  $H > M > L$ , and the signal is binary. Suppose that  $H$  types are much more likely to be associated with one signal while  $L$  types with the other, while  $M$  types are associated with either with equal probability. In such a situation,  $H$  is sufficiently confident that she will be told apart from  $L$  and hence she is more interested in distinguishing herself from  $M$ . As  $M$  is signalling to distinguish himself from  $L$ , the way for  $H$  to be different is by not signalling.

heart agency but choose to search by patronising nightclubs are much more likely to meet a high quality potential partner, since the choice of nightclubs or similar venues is clearly not random: dress codes, bouncers, high prices for drinks are all imperfect substitutes for membership fees. This reduces the opportunity cost of not joining, reducing further the non-joiners' incentive to join. Moreover, if the intermediary has access to a technology less perfect than what we have assumed here, then again joining it clearly becomes less attractive. A countervailing tendency is the fact that, while we have assumed that matching occurs with probability 1 both through the intermediary and in the direct market, in practice the frequency of matches would be lower in the direct market.

We note however that a technological advantage by the intermediary is essential for the existence of sandwich equilibria. Suppose matching through the intermediary is in fact random. It can be shown that, if  $c$  is positive, there is no sandwich equilibrium. When  $c = 0$ , there is a unique sandwich equilibrium:<sup>16</sup> this is given by the condition that the average type is the same in the direct and the intermediated markets. Because of supermodularity, for a strictly positive fee different types would be indifferent at different differences in averages, so only threshold equilibria can exist.

**4.4.** A natural question of interest is the shape and size of the set of sandwich equilibria. This matters because, if it turned out in practice that any sandwich equilibrium is arbitrarily close to a threshold equilibrium, which is a degenerate  $\{\underline{t}, 1\}$ -sandwich equilibrium, then our analysis would not be able to explain the observed regularity which motivate it, that “a lot” of top people do not use intermediation. While characterising in general the set of sandwich equilibria is hard, a simple example suffices to address this question. In Figure 3,<sup>17</sup> we illustrate the entire set of sandwich equilibria, in the special case of a

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<sup>16</sup>Provided we maintain the assumption of assortative matching in the second period.

<sup>17</sup>The appendix (available on request or at [sites.google.com/site/giannidefraja/](http://sites.google.com/site/giannidefraja/)) provides

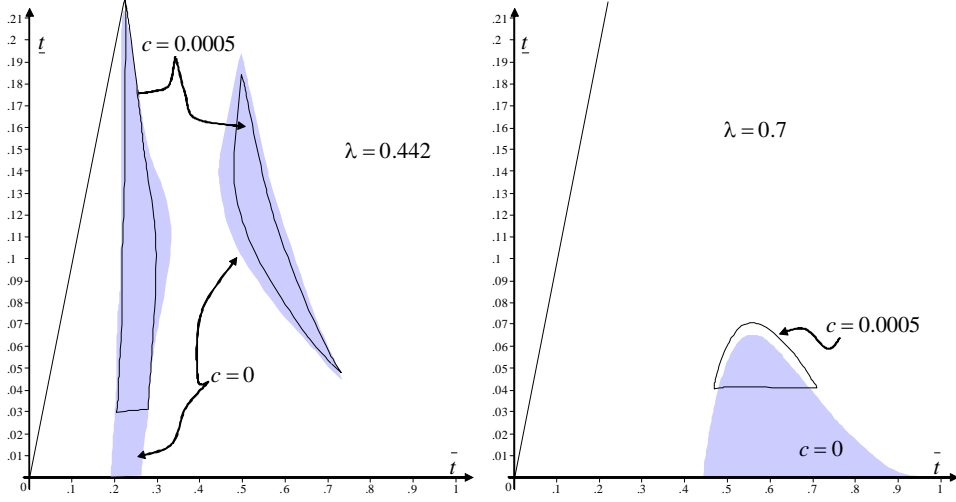


Figure 3: The sandwich equilibria for two values of  $\lambda$  and two values of  $c$ .

uniform density function for  $v$  and  $q$ , that is when  $f(t) \equiv 1$ . The two diagrams depict the lower portion of  $[0, 1]^2$  (note that the diagonal is not a 45 degree line, as the axes are drawn in different scales). The highlighted regions are the sets of sandwich equilibria for two values each of  $c$  and  $\lambda$ . In both diagrams, the shaded (respectively outlined) area represents combinations of points  $(\bar{t}, \underline{t})$  such that a  $\{\underline{t}, \bar{t}\}$ -sandwich equilibrium exists when  $c = 0$  (respectively  $c = .0005$ ).

For  $c = 0$  and  $\lambda > 1/2$ , the set of sandwich equilibria is drawn on the right hand side of Figure 3: the lower contour of the set is the set of points on the horizontal axis between a critical value of  $\bar{t}$  and 1; the upper contour of the sandwich equilibrium set is a locus strictly above 0 for  $\bar{t} < 1$ , which tends to 0 as  $\bar{t}$  tends to 1. When  $c$  is positive, clearly there cannot be equilibria where the lowest types join the intermediary as they cannot afford the fee: but note the “multiplier” effect of the fee, it is not the case that types just below the lowest type who join, are deterred from joining by the fee (which is tiny fraction of

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details of how the pictures have been obtained. In essence, it was a “brute force” process: given  $\underline{t}$  and  $\bar{t}$  we checked that no trader in  $[0, 1]$  had an incentive to deviate, and repeated the procedure on a fine grid, checking for every possible pair of points  $(\bar{t}, \underline{t})$  below the diagonal.

their valuation); rather, if there are too few low types joining then the highest types who should join the intermediary will prefer to deviate as they are likely to be matched with types from the upper end of the distribution.

Threshold equilibria are points on the vertical segment joining  $(1, 0)$  and  $(1, 1)$ . By Proposition 2, all points on a vertical segment with its lower end at  $(1, 0)$  are threshold equilibria, and, for  $c = 0$ , by Corollary 2 the set of threshold equilibria is the entire segment joining  $(1, 0)$  and  $(1, 1)$ . Note therefore that, for  $c = 0$ , the equilibrium set (the union on the set of sandwich and of threshold equilibria) is connected for high  $\lambda$  and disconnected for low  $\lambda$ . Numerical simulations suggest that, for the uniform density case we considered the cut-off value of  $\lambda$  is  $\frac{1}{2}$ . As Proposition 3 gives sufficient conditions, it is not surprising that there exist sandwich equilibria for  $\lambda < 1/2$  as well. Note that, as shown in Figure 3, in the low  $\lambda$  case,  $\bar{t}$  can be very low, and even for  $\lambda$  high there are sandwich equilibria where no trader above the median uses the intermediary.

## 5 Concluding remarks

This paper contributes to our understanding of markets where intermediaries are active but traders may also choose to trade unassisted.

Stylised facts indicate that trader behaviour may be non-monotone. For example, consider education: the widespread evidence of assortative matching in the marriage market (eg. Blossfeld and Timm, 2003) indicates that it is a supermodular characteristic: more educated individuals value education more. With this in mind, consider the diagram in Figure 4. It is an illustration derived from the dataset constructed by Hitsch et al (2010). They study the link between mate preference and match formation, and report data on the education level of internet users at large and a representative sample of the members of a major online dating service provider in San Diego and Boston. In the picture, we choose convenient thresholds in their ranking to determine

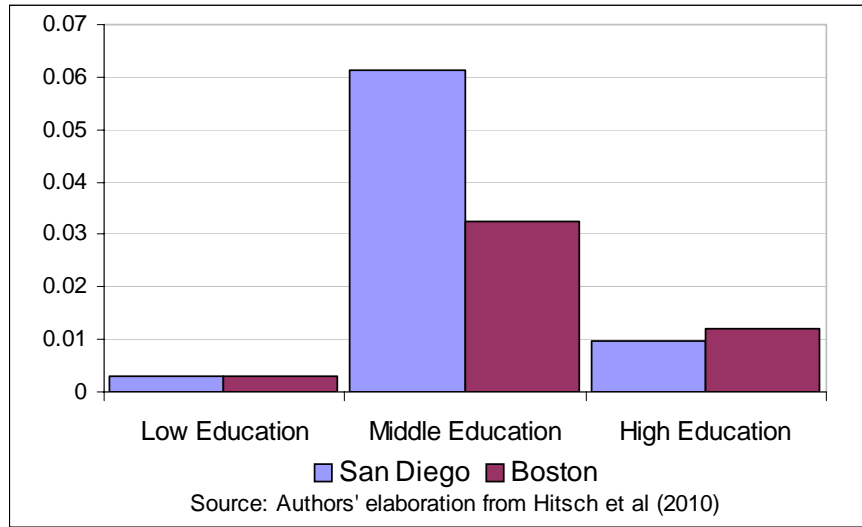


Figure 4: Probability that an internet user looks for a partner in the dating agency

three groups. The figure plots the percentage of internet users belonging to each group who have joined the dating service. The diagram suggests a non-monotonic relation between “type” and the propensity to join the intermediary: those with intermediate education levels are considerably more likely to use the online provider than types with higher or lower education.<sup>18</sup>

In this paper we have rigorously established that this non-monotone behaviour, where the traders who have the most to gain from using the superior matching mechanism provided by the intermediary choose not to use it, may indeed form part of an equilibrium. The existence of such “sandwich” equilibria, however, is subject to the strong condition that the traders’ continuation value is sufficiently increasing in their type. Consequently the intertemporal characteristics of each market determine whether or not sandwich equilibria can

<sup>18</sup>The diagram is only suggestive, as clearly only a small fraction of the internet users are looking for a partner in the period considered. It seems plausible that the likelihood of being looking for a partner is roughly independent of education. In this case, if we conditioned the joining probabilities on actually being looking for a partner, the height of each bar would increase roughly proportionally, and their relative size would not change.

exist: high type traders have most to gain from trade now, this must continue to be the case if they choose to delay trade.

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